Comments on the paper Time measurement using a realizable atomic clock

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Comments on the paper *Time measurement using a realizable atomic clock*

Abstract. A recent statement of L. M. Stephenson in connection with complete clocks is disputed and the traditional time dilatation of the special theory of relativity is upheld.

Special relativity is a selfconsistent theory well in agreement with existing experimental data. Fundamental results of the theory cannot be modified without seriously affecting the entire theory. Bearing this in mind, one is surprised by the efforts of various authors to disprove some fundamental results of the theory. One of such attempts is a recent contribution of Stephenson (1970). This author, as some others (Dingle 1956, 1967, 1968, Pardy 1969), claims that the familiar time dilatation does not apply to complete clocks (as opposed to atomic or subatomic particles). By a complete clock a device is understood consisting of two parts: an oscillating device that should take care of the time unit and an integrating device that should be connected with the measurement of time lapse. Cases in which both parts are in different inertial reference frames lead to the familiar time dilatation. The crucial statement appears as both devices are put together to a complete clock in one inertial frame (the 'moving' frame S′) and its reading is compared with an identical clock in another inertial frame (the 'rest' frame S). The statement that the length unit used by S′ is smaller than that used by S contradicts the relativity principle according to which the laws of nature are equally valid for all inertial frames.

Each inertial observer is entitled to define his units of length, time... . If all observers agree to define them in the same way in terms of the wavelength of the krypton spectral line, of the period of the caesium electromagnetic radiation... according to the relativity principle these units for each observer in his frame are exactly the same. However, two different inertial observers in comparing their proper length or time measurement with the measurement of the corresponding unproper length or time by the other observer encounter length contraction (if the length is parallel to the relative velocity) or time dilatation. So one has in Stephenson's case considering a variant of the Feynman clock (e.g. Sears 1963), $L' = L$ if L' is the length of the resonator stationary in S′ as measured in S′ and L is the length of the resonator stationary in S as measured in S. If the unit of length should transform as $L' = L(1-v^2/c^2)^{-1/2}$ all inertial frames would not be equivalent; the frame S would be privileged. Furthermore, the above equation applies to the length parallel to the relative velocity v of S′ and S only. So for the observer in S′ the unit of length would be direction dependent. This violates the assumption of space and time homogeneity which is inherent in the theory.

A further question may arise if at first two identical measuring devices, for example atomic clocks, are both in the same inertial frame. Are the devices still equivalent or ideal after the transition to different inertial frames? This means: are they identical from the viewpoint of observers that are at rest in respect to the devices? The answer is yes if the internal states of the devices are not altered during acceleration. But the internal states remain unaltered if the acceleration does not exceed an upper limit. So it is possible to give a measuring device an arbitrary high velocity (less than c) relative to the other device if the acceleration remains low. This is a theoretical prediction as well as an experimentally established fact (eg Bondi 1957).

The unobservability of length contraction of moving bodies cannot be brought in by analogy with the absence of time dilatation. Indeed, it was proved (Terrell 1959) that rapidly moving objects would not appear longitudinally contracted but rather rotated around an axis perpendicular to the direction of motion and to the direction of observation. But this has nothing in common with time dilatation since the time part of four-dimensional space–time is only one dimensional (Dorling 1970).

Stephenson's final argument reads as follows: a complete clock is measuring the number of ticks (signals) that should be the ratio of the time interval between a given pair of events and the time interval between ticks. Since both time intervals should be transformed according to the same transformation their ratio should be invariant. Thus the measurement corresponding to a given pair of events by a complete clock should be the same in any inertial frame. It is not difficult to disprove this statement. First of all, the time interval between ticks, that is, the unit of time, is equal for any inertial observer, as discussed above. Also some counter arguments have been promoted in connection with Dingle's discussions (Darwin 1957, McCrea 1967). Nevertheless the number of signals was not considered explicitely so it may be worthwhile to consider it in some detail.

By convention electromagnetic waves have to be used in special relativity for the transmission of information between observers. The frequency of an electromagnetic wave is not transformed, however, as reciprocal time. Rather it is transformed according to

$$\nu' = \frac{\nu(1-v/c)^{1/2}}{(1+v/c)^{1/2}}$$

if ν is the frequency for the observer at rest with respect to the source and ν' is the frequency for the observer receding from the source with velocity v. The general formula

$$\nu' = \frac{\nu(1+v\cos\theta/c)}{(1-v^2/c^2)^{1/2}}$$

has been experimentally verified up to terms quadratic in v/c (Mandelberg and Witten 1962).

Two atomic clocks with proper frequency ν should be equipped with transmitter and receiver. The first clock and the corresponding observer are situated at the origin O of the intertial frame S. The second clock and the corresponding observer are situated at the origin O′ of the frame S′. The origin O′ is moving with uniform velocity v in S along the x axis which coincides with the x' axis. At the event A ($x = 0$, $t = 0$) as both origins coincide the clocks are synchronized. At the event B ($x = X = vT$, $t = T$) the origin O′ encounters a given space point in S. According to the Lorentz transformation

$$x' = (1-v^2/c^2)^{-1/2}(x-vt), \qquad t' = (1-v^2/c^2)^{-1/2}(t-vx/c^2)$$

the events A and B are described from the viewpoint of the observer in S′ as ($x' = 0$, $t' = 0$) and $x' = 0$, $t' = T' = (1-v^2/c^2)^{1/2}$ respectively. Between the events A and B the observer in S counts $\nu T = N$ signals from his proper clock and

$$\nu' T = \frac{(1-v/c)^{1/2}N}{(1+v/c)^{1/2}}$$

signals from the clock in S′. Between the same events the observer in S′

counts $\nu T' = (1-v^2/c^2)^{1/2}N$ signals from the proper clock and $\nu' T' = (1-v/c)N$ signals from the clock in S. The proper frequency is the same for both observers as the clocks are identical and in the same internal state; so $1/\nu$ can be used to define the unit of time by both of them. Obviously, the number of signals (or ticks) is not invariant. The difference between the number of signals from the proper clock and from the unproper clock is equal to

$$\left(1-\frac{(1-v/c)^{1/2}}{(1+v/c)^{1/2}}\right)N.$$

for the observer in S and to

$$\left(1-\frac{v^2}{c^2}\right)^{1/2}\left(1-\frac{(1-v/c)^{1/2}}{(1+v/c)^{1/2}}\right)N$$

for the observer in S′.† All these results agree with the usual interpretation of the time dilatation. There is not a single argument against the time dilatation.

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Time measurement: Criticism of a paper by L. M. Stephenson

Abstract. A recent paper on time measurement by L. M. Stephenson is examined in detail. It is argued that the paper sheds no new light on the subject and that it is altogether misleading. In particular, while the notion of a primary time scale in special relativity bears some relation to certain elements that are familiar and valid, Stephenson's own treatment of the notion appears to be wholly erroneous.

† Both frames are treated on an equal footing but the equations are not completely symmetrical to $v \to -v$, $S \to S'$, $S' \to S$. To acquire complete symmetry the discussion should be repeated starting from the pair of events ($x' = 0$, $t' = 0$) and ($x' = -vT'$, $t' = T'$) in S′.